

Sequential Direct and Inverse Modeling of Underground Flows in the Upper Cheliff Alluvial Aquifer, Algeria

Modellazione seguenziale diretta e inversa di flusso delle acque sotterranee nell'acquifero alluvionale della Piana del Cheliff Superiore, Algeria

Ourdia KABEN^a 🖆 , Djamel MAIZI^a, Mebarka TAKORABT^a

^a Université des Sciences et de la Technologie, Houari Boumediene, B.P. 32, 16123 Bab-Ezzouar, Algiers, Algeria e-mail 🝯 : kaben_m@yahoo.fr

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Correspondence to: Ourdia Kaben 🖆 kaben_m@yahoo.fr

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Riassunto

La Piana del Cheliff Superiore si trova 150 km a sud-ovest di Algeri, capitale dell'Algeria. È caratterizzata da un acquifero non confinato costituito da depositi eterogenei di età Mio-Plio-Quaternaria. Attualmente l'acquifero è intensamente sfruttato per soddisfare i bisogni domestici, industriali ed agricoli della regione.

L'uso della modellazione numerica per la gestione delle risorse idriche sotterranee è cruciale al fine di migliorare la conoscenza dell'idrogeologia di questo acquifero. In questo lavoro abbiamo sviluppato un approccio basato sulla risoluzione in seguenza del problema diretto ed inverso. Nel caso dell'approccio diretto, le simulazioni di flusso in stato stazionario hanno mostrato la persistenza di ampie discrepanze tra i valori piezometrici osservati e simulati, che riflettono l'eterogeneità dell'acquifero. La zonazione ottenuta attraverso l'approccio diretto, basata sul modello concettuale, è stata utilizzata come input nell'applicazione del metodo inverso. Quest'ultimo, in aggiunta alla velocità di computazione ed alla convergenza, ha permesso di ridurre le differenze tra le misure dei livelli piezometrici osservati e simulati.

I principali risultati evidenziano una distribuzione eterogenea della conducibilità idraulica, i cui valori più elevati sono stati registrati nelle zone centrali e nord-occidentali dell'acquifero, mentre i valori più bassi nelle zone settentrionali, orientali e meridionali dell'acquifero. Inoltre, il calcolo del bilancio di flusso ottenuto dal modello inverso mostra che la ricarica dovuta alle precipitazioni costituisce la principale entrata nell'acquifero con 462.5x10³ m³/giorno, mentre la principale uscita è rappresentata dal Wadi Cheliff, pari a 627x103 m3/giorno.

Abstract

The Upper Cheliff plain lies 125 km south-west of Algiers, capital city of Algeria. It is characterized by its unconfined aquifer contained in a heterogeneous filling of Mio-Plio-Quaternary age. Nowadays, this aquifer is severely exploited to meet the domestic, industrial and agricultural needs in the region.

Groundwater resources management by means of numerical modeling is highly required in order to improve our knowledge of the bydrogeological operation of this aquifer. In this work we implemented an approach based on a sequential solving of the direct and inverse problems. In the case of the direct problem, steadystate flow simulations revealed the persistence of large discrepancies between the measured and calculated piezometric levels, which reflect the heterogeneity of the aquifer. The zonation provided by the direct method, based on the conceptual model, is then fed in the inverse method. The latter, in addition to its speed and its convergence, made it possible to reduce the disparities with measurements as noted in the direct method.

The main results highlight a heterogeneous hydraulic conductivity distribution which reaches its highest values in the central and north-western sectors of the aquifer, whereas the lowest values are found in the eastern, northern and southern sectors of the aquifer. Furthermore, calculation of the groundwater flow budget by the inverse model showed that rainfall recharge is the main input into the aquifer with $462.5 \times 10^3 m^3/day$, whereas the most important outlet of the aquifer is the Cheliff Wadi with a flow rate of $627 \times 10^3 m^3/day$.

Introduction

The Upper Cheliff alluvial aquifer represents a very important water reservoir that has been intensively exploited in recent years to meet domestic, industrial and agricultural needs. To overcome this situation, aquifer management supported by numerical modeling becomes necessary to guide a sustainable exploitation of the resource. The implemented approach relies on a sequential resolution that starts with the conceptual model-based direct method and moves on to a refined resolution that uses the inverse problem and zonation provided by the initial direct method. The traditional trialand-error method is the most extensively used technique for calibrating underground flow models (Anderson & Woessner, 1992). The spatial distribution and hydraulic conductivity of the porous media are manually changed in this procedure until a satisfactory match between the estimated and measured piezometric levels is achieved at each observation point.

However, this manual method is proved to be time consuming, tedious, subjective, and largely dependent on the experience of the hydrogeologist, which has prompted researchers to develop automated calibration techniques. The aforementioned technique also described as an inverse modeling technique has been intensively studied over the past forty years. The relevant literature is vast and diverse, such as (Carrera & Neuman, 1986a; 1986b; 1986c; Carrera et al., 2005; McLaughlin & Townley, 1996; Vrugt et al., 2008; Yeh, 1986) provide comprehensive summaries of the many existing methods for solving the inverse problem in hydrogeology. In this context, our work is devoted to determining the hydraulic conductivity of the Upper Cheliff alluvial aquifer in steady-state flow conditions, where only stabilized piezometric levels are available. The direct problem-solving is done by the deterministic conceptual model approach that allows the construction of an efficient model representation. This approach shows, after some manual adjustment, the existence of significant differences between the measured and calculated piezometric levels. Taking the anomaly of the calibration model, the complexity of the hydrogeological reservoir, and the lack of data into consideration, the original objective was adjusted by addressing the inverse problem. This involves finding model parameters that minimize the discrepancy between observed and estimated values, resulting in model calibration.

The approach derived from maximum likelihood estimation, which employs the zonation technique, was used to achieve this resolution (Carrera & Neuman, 1986a; 1986b; 1986c; Cooley, 1977). The two aforementioned methods made it possible to assess the aquifer water budget as well as fine-tune the geographic distribution of the hydraulic characteristics.

Materials and Methods *The study area*

The Upper Cheliff plain is located north of Tell, between the Boumaâd mountain and the Ouarsenis foothills. As shown in Figure 1, the aquifer, is located between latitudes



Fig. 1 - Geographical location of the Upper Cheliff plain. Fig. 1 - Ubicazione geografica della Piana del Cheliff Superiore.

 $36^{\circ}6^{\circ}-36^{\circ}18^{\circ}$ N and longitudes $2^{\circ}-2^{\circ}27^{\circ}$ E. It consists of a subsidence ridge that extends east to west and is more than 60 km long and 25 km wide on average.

The plain is characterized by a semi-arid climate influenced by the Sahara Desert in summer and the Mediterranean Sea in winter, characterized by two distinct seasons: wet and cold winters and hot and dry summers (Touhari, 2015).

The average monthly climate balance illustrated in Figure 2 highlights two opposite periods:

- A wet period observed from December to May where the average monthly potential evapotranspiration (ETP) is lower than the monthly rainfall (P) and the rainy maxima occur in March. Indeed, the wet months provide significant surpluses of water to the tributaries and favor the recharge of groundwater.
- A dry period from May to October with the highest evaporation demand recorded in July and August, causing aridity and a gradual decrease in soil water reserves.

From the lithostratigraphic point of view, the Upper Cheliff watershed was formed as a landmass entirely from the Mio-Plio-Quaternary period. As illustrated in Figure 3, Upper Cheliff alluvial aquifer is essentially characterized by coarse alluvium and pebbles in the central sector of the valley with variable thicknesses ranging from 50 m to 145 m, and a layer of clay and silt covering the coarse alluvium in the south-wester sector of the valley with thicknesses ranging from 7 m to 20 m. This alluvial aquifer overlays a gravelly clay layer or a marl bedrock, representing aquitards/aquicludes (Mattauer 1958).



Fig. 2 - Seasonal evolution of the monthly average climatic balance in the Upper Cheliff aquifer (1973-2013).

Fig. 2 - Andamento stagionale del bilancio climatico medio mensile dell'acquifero della Piana del Cheliff Superiore (1973-2013).

Methodology

Modeling groundwater flows in porous media involves first identifying boundary conditions and investigating hydrodynamic parameters, and then setting up sequential simulations. Modeling is accomplished by assessing available data and measurements collected in the study area. We should remind out that the major characteristics that describe an aquifer are typically known only at some locations dispersed



Fig. 3 - Geological map of the studied area with wells and piezometers location (a) and a SSE-NNW hydrogeological section, extracted from the hydrogeological map of Algiers region drawn up in 1973 by the National Institute of Cartography, Algeria (b).

Fig. 3 - Carta geologica dell'area di studio con l'ubicazione di pozzi e piezometri (a) e sezione idrogeologica orientata SSE-NNW, ottenuta dalla carta idrogeologica della regione di Algeri, realizzata dall'Istituto di Cartografia Nazionale Algerino nel 1973 (b).

Tab. 1 - Monthly precipitation of the Upper Cheliff basin for 1990.

Tab. 1 - Precipitazione mensile nel bacino del Cheliff Superiore nel 1990.

Month	Sep.	Oct.	Nov.	Dec.	Jan.	Fab.	Mar.	Apr	May	Jun.	Jul.	Aug.	Total
Rainfall (mm)	19.6	18.1	43	63.6	51.0	69.6	88.7	20.0	13.4	8.1	1.9	8.4	395.8

over the whole research region and may be inaccurate. As a result, we attempt to estimate these parameters throughout the whole domain using just incomplete knowledge. Kriging aids in understanding the spatial organization of the phenomena and determining the features of the underground flow as precisely as possible.

Annual cumulative rainfall for 1990 is estimated at 395.8 mm. Table 1 shows a wet period from November to March and a dry period from April to September. The amount of precipitation falling during the wet period accounts for about 80% of the annual precipitation.

The measurement of the piezometric levels carried out in May 1990 in 70 wells and boreholes allowed to reconstruct the piezometric surface for the Mio-Plio-Quaternary aquifer. The interpolation of piezometric data through kriging technique revealed a preferential groundwater flow axis overlap on eastwest direction of the valley (Fig. 4). This is the piezometric reference map we have used for modeling underground flows.

It is critical to identify the nature of the system's boundary conditions in order to better comprehend the flux exchange between the discrete domain and the external environment. The modeled aquifer area is characterized by watertight boundaries which are affected:

- To the northwest and to the south at the edge of the impermeable Cretaceous shales;
- To the northeast where the piezometric curves are perpendicular to the Gantas massif.

The studied region is discretized horizontally into 387 m x 417 m rectangular cells distributed according to a matrix of 50 lines and 100 rows, with 2098 active cells covering an area of 338.57 km². The vertical discretization is carried out using a monolayer that is limited at the top by the natural land and at the bottom by an impermeable layer. The baseline limit of clay-formed aquifers was determined using data from 24 boreholes reaching back to the Pliocene.

Considering the porous lithology of the land outcrops (alluvium, sand, etc.) and the very low slopes of the plain, recharging is mostly due to rainfall infiltration. The sole means by which the aquifer loses water are through drainage into wadis and pumping, mostly for drinking water distribution. Evapotranspiration is expected to be modest due to the depth of the water table, which is frequently higher than two meters, whereas the vegetation cover is low, even negligible. Unfortunately, the hydraulic conductivity data does not reflect reality on the field. In fact, the report released by the Algerian National Agency for Hydraulic Resources (ANRH 2004) contains too little material to address the entire subject under consideration. Thus, we have used the transmissivity and thickness maps from the hydrogeological investigations made by Mania and Djeda (1990). By overlapping the two

Modeling of flows by the conceptual approach

The conceptual model is defined as a physical and/or chemical representation of the processes that affect or govern an actual aquifer system (Anderson & Woessner, 1992; ASTM, 1995a; 1995b; lstok, 1989). The main purpose of conceptual modeling is to simplify ground complexity by integrating, in an organized way, the collected data, which allows consequently faster and less complex analysis. This simplification is necessary since the complete reconstruction of the aquifer system is impossible (Anderson & Woessner, 1992).

In a saturated porous medium, water flows can be described by laws involving parameters related to the aquifer geometry, its hydrodynamic characteristics and external stresses. The corresponding equations, reduced to two horizontal dimensions, are as follow (Bear, 1979):

Darcy's law for incompressible fluids:

$$V_i = K_{ij} \frac{\partial h}{\partial x_i} \tag{1}$$

Where V_i is the Darcy velocity or flux, K_{ij} is a permeability tensor component and h is the piezometric head. The effective flow velocity mean is given by $v_i = V_i/n_e$, where n_e is the effective porosity linked to the mobilizable water reserve.

The diffusivity equation:

$$\frac{\partial}{\partial x_i} \left[T_{ij} \frac{\partial h}{\partial x_i} \right] = S \frac{\partial h}{\partial x_i} + q \qquad (2)$$

Where $T_{ij}=bK_{ij}$ is a component of the transmissivity tensor, q is the source term representing infiltration (negative) or withdrawal (positive), S is the storage coefficient and b is the saturation thickness of the aquifer.

Inverse modeling

The goal of automatic calibration, also known as inverse modeling, is to find the optimal parameter reproducing realworld data by means of inverse problem numerical simulations and optimization techniques. In this work, inverse problem solving is carried out by means of an approach derived from the maximum likelihood estimation associated to the zonation method as described by (Carrera & Neuman, 1986a; 1986b; 1986c; Carrera et al., 2005).

Nonlinear regression employing the Gauss-Marquardt-Levenberg method is used to optimize the parameters throughout minimizing the objective function. This approach attempts to minimize, in the sense of least squares, the difference between the estimated and the measured (i.e., observed) piezometric levels. Mathematically, this results in minimizing the objective function ϕ defined by:

$$\phi = \sum_{i}^{N} W_{i} \left(c_{i} - c_{i}^{'} \right)^{2}$$
(3)

Where W_i is the weight assigned to the ith measure, c_i is the ith numerical generated observation, c'_i is the ith experimental observation and N is the total number of observations. Note that the studied domain is subdivided into multiple zones, each of which is given an unknown parameter representing either hydraulic conductivity or recharging. These parameters, to be estimated, are considered uniform over each zone and their number therefore equal to the number of zones.

Results and discussion

We performed a preliminary calibration by "trial and error" in order to constrain automatic calibration as much as possible. This provided us the ability to specify the hydraulic conductivity polygons. Our choice to use a conceptual model was motivated by the need for high accuracy as well as the quantity and quality of data that were available. Checking whether the model can reproduce the hydraulic heads observed on the ground using the input parameters and boundary conditions is done through the flow model calibration step, which consists of adjusting the flow model to existing piezometric data.

Conceptual model calibration

The manual calibration technique known as "trial and error" consists of adjusting the model parameters until the difference between observed and calculated (i.e., simulated) values is minimized to achieve a satisfactory calibration. The best calibration results are shown in the following figures (Fig. 4, 5, 6) after multiple readjustments of the hydraulic conductivity and the recharge.



Fig. 4 - Comparison of measured and calculated piezometric levels at steady state (year 1990).



First, we overlaid measured and calculated piezometric levels, in a steady state flow condition after calibration. These results reveal a good reconstitution of the piezometric level and a flow axis showing a general east-west direction (Fig. 4).

The calibration of the model associated to the geographical distribution of the observation points is shown in Figure 5, in which the residual values are calculated between calibrated and measured piezometric levels. We note that the minimum residual value is 0.05 m (well 17) while the maximum value is 2.48 m (well 57).



Fig. 5 - Difference between measured and calculated piezometric level after trial-anderror preliminary calibration.

Fig. 5 - Differenza tra i livelli piezometrici osservati e simulati successivamente alla calibrazione preliminare (*trial-and-error*).

Figure 6 depicts the distribution of hydraulic conductivity as a result of the model's calibration in steady state. It indicates nine permeability zones with a range of 125 m/day to 5 m/day which help to understand the aquifer's material variability. While the low values are found in the south, north, and east sectors of the plain, the high values are found in the central and northwestern sectors. It appears that high hydraulic conductivity values are found in regions with low hydraulic gradients, which might provide us with information about the aquifer's flow.



Fig. 6 - Distribution of hydraulic conductivity simulated by direct modeling in steady state.

Fig. 6 - Distribuzione della conducibilità idraulica simulata ottenuta mediante l'approccio diretto in stato stazionario.

Inverse model calibration

We have once more used piezometric values of May 1990, as a reference in this section of automated calibration. We employed the zones to parameterize hydraulic conductivity and recharge. The initialization of these zones is made using values derived from the conceptual model's calibration.

Figure 7 shows the piezometric map generated by the automatic calibration and the observation points where the residual values are calculated after calibration. We note an improvement in the calibration through targets with a minimum residual value of 0.01 m (well 26) and a maximum value of 1.51 m (well 6).



Fig. 7 - Distribution of bydraulic conductivity simulated by direct modeling in steady state. Fig. 7 - Differenza tra i livelli piezometrici osservati e simulati successivamente alla calibrazione mediante modellazione inversa in stato stazionario.

The piezometric curves of the Mio-Plio-Quaternary aquifer show variable spacings in relation to reservoir geometry and permeability. In the central sector of the plain, the very low hydraulic gradient are associated to low conductivity as well, which indicates a weak flow-regime.

Figure 8 illustrates the hydraulic conductivity, ranging from a maximum of 150 m/day to a minimum of 7 m/day. These results are close to those produced by calibration through trial and error, as seen in Figure 6. This is most likely because several manual simulations were run to refine the parameters.



Fig. 8 - Distribution of bydraulic conductivity simulated by inverse modeling in steady state.
Fig. 8 - Distribuzione della conducibilità idraulica simulata ottenuta mediante modellazione inversa in stato stazionario.

It should be mentioned that this method is quite useful since the parameters are determined automatically and modeling time is significantly reduced.

It is worthy to be mentioned that initialization settings are crucial for ensuring quick algorithm convergence and improved agreement between observed and computed values. Thus, calibration of the spatially variable hydraulic conductivity was done iteratively over 11 iterations and presented a Weighted Mean Residual of 0.21.

Figure 9 depicts the distribution of the recharge as determined by model calibration and demonstrates that:

- The highest values are located in the North-West and South-East sectors of the plain which are at the origin of the nature of the alluvial soil and the rise of the substratum,
- The lowest recharges are located in the center to the southern sector of the plain.



 Fig. 9 - Simulated recharge distribution by inverse modeling in steady state.
 Fig. 9 - Distribuzione della ricarica simulata mediante modellazione inversa in stato stazionario.

We then evaluated the correlation between the measured and the simulated groundwater levels. As shown in Figure 10, we can confirm a very good agreement between the simulation results and the observation points as we calculate a correlation coefficient $R^2 = 0.998$.



Fig. 10 - Correlation between observed values and calculated values. Fig. 10 - Correlazione tra i valori di carico idraulico osservati e simulati.

Once the model is calibrated, we can calculate the flow balance budget of the aquifer. Table 2 gives the flux values calculated by the steady-state model in 1990.

Tab. 2 - Budget sheet of exchanged flows of the Mio-Plio-Quaternary aquifer in steady state (1990).

Tab. 2 - Bilancio di flusso per l'acquifero Mio-Plio-Quaternario in stato stazionario (1990).

Budget term	Outputs (m ³ /day)	Inputs (m³/day)		
Wells	19678	0		
River leakage	627230	184424		
Recharge	0	462481		
Total	646908	646906		

The flow budget allowed us to check the correct model calibration by calculating the convergence. In our case, the simulation deviation, in percent discrepancy, is only 2.8×10^{-4} %. Note that rainfall recharge is the main entry into the aquifer with a flow rate of about 462×10^3 m³/day. The most important outlet of the aquifer is the Cheliff Wadi with a flow rate around 627×10^3 m³/day.

Sensitivity analysis

The model's calibration parameters, such as the hydraulic conductivity and the recharge, are subjected to a sensitivity analysis. By changing the parameters, the model's sensitivity was examined in relation to how it affected the piezometry.

The parameters sensitivity diagram of Figure 11 shows that:

- The less sensitive areas to fluctuations in hydraulic conductivity are located to the west and south of the plain, while the most sensitive areas occupy the rest of the plain with a sensitivity coefficient up to 0.9.
- The most sensitive areas to fluctuations in recharge are found in the northwest and southwest of the plain.

According to the study of the sensitivity to the parameters, it can be deduced that the western part of the study area reacts in the same way with respect to changes in the two parameters.

Conclusion

Numerical modeling of the Upper Cheliff aquifer has allowed us to better understand the hydrodynamic operation of the aquifer. Numerical simulations in permanent regimes



Fig. 11 - Hydraulic conductivity (HK) and recharge (RCH) distributions (a and b) with parameter sensitivity (c) using groundwater level as simulated variable.

Fig. 11 - Distribuzione della ricarica (RCH, a) e della conducibilità idraulica (HK, b) e analisi di sensitività utilizzando i valori dei livelli piezometrici come variabile.

carried out by i) the trial-error method and ii) the automatic calibration method made it possible to refine the spatial distribution of the hydraulic characteristics and to evaluate the water budget of the aquifer.

The first method consists in simulating the flow using the conceptual model based on manual calibration, which turns out to be long and tedious. We noted that large disparities between the estimated and observed hydraulic head values still exist, in spite of the periodic adjustments of the calibration parameters, which can be attributed to the aquifer's heterogeneous lithology.

The second strategy, which is based on maximum likelihood estimate, entails inversely modeling the flow. This method, which searches for parameters using the zonation approach, has shown to be quick and automated. It is crucial to keep in mind that this technique makes use of the recharge findings from the preceding conceptual model as well as the initial values for the hydraulic conductivity polygons. This technique outperforms the previous one on globally, which results in a significant decrease in the discrepancies between measured and calculated hydraulic heads.

Furthermore, the conductivity distribution map showed that the high values, which reach up to 150 m/day, are located in the central and northwest sectors, while the low values are found in the east, north and south areas of the plain with a minimum of 7 m/day.

The calculation of the water flow budget by the inverse model showed that the recharge by rainfall constitutes the main entry into the aquifer, whereas the most important outlet of the aquifer is the Cheliff Wadi.

A sensitivity analysis was carried out to identify the areas that were impacted by parameter changes. It showed how the western part of the study region reacts to changes in hydraulic conductivity and recharge in a similar way.

To conclude, before employing automatic calibration, there must be a manual calibration step. This operation allows the technique to converge and allows the choice of reasonable starting values. Lastly, as the calculated parameters provide a sufficient database for numerical modeling, the aquifer should be studied in a transient regime in order to understand the combined effects of the decrease in rainfall and the intensive exploitation in recent decades, which are responsible for the general decline of the piezometric levels.

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Author contributions

This work is part of Ms. Kaben's PhD thesis, carried out under the supervision of Mr. Maizi. Therefore, Ms. KABEN conducted the research, material preparation, data collection and analysis. Ms. Takorabt contributed and assisted in the creation of maps.

Ourdia KABEN wrote the original version of the paper, and other authors contributed to its revisions and improvements.

The final version of the paper has been read and approved by all authors.

Competing interest

The authors declare no conflict of interest.

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Additional information

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