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From Fourier to Darcy, from Carslaw to Theis: the analogies between the subsurface behaviour of water and heat

Da Fourier a Darcy, da Carslaw a Theis: le analogie del comportamento delle acque e del calore nel sottosuolo

David Banks

Riassunto: L'analogia tra il comportamento dell'acqua sotterranea in un acquifero e il comportamento conduttivo del calore in un estifero è quasi perfetta. Lo sviluppo della teoria del flusso delle acque sotterranee al fine di determinare le soluzioni matematiche si è basato fondamentalmente sulle analoghe leggi termiche: la Legge di Darcy è analoga a quella di Fourier. Charles Theis sviluppò la sua analisi del flusso radiale delle acque sotterranee basandosi sulle equazioni di conduzione di Horatio Scott Carslaw. Ogata e Banks si riferiscono al lavoro sulla diffusione del calore di Carslaw e Jaeger, per risolvere la dispersione longitudinale dei soluti nelle acque sotterranee. L'idrogeologo è quindi dotato perfettamente dei necessari modelli concettuali, strumenti matematici e codici numerici per diventare un praticante "termogeologo" professionista

Parole chiave: Idrogeologia generale, termogeologia, idraulica delle acque sotterranee, storia dell'idrogeologia, conduzione del calore

Keywords: *General hydrogeology, thermogeology, groundwater hydraulics, history of hydrogeology, heat conduction*

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Abstract: *The analogy between the behaviour of groundwater in an aquifer and the conductive behaviour of heat in an aestifer is almost perfect. The development of groundwater flow theory has consistently drawn upon thermal analogues to find mathematical solutions: Darcy's Law is analogous to Fourier's. Charles Theis drew on Horatio Scott Carslaw's heat conduction equations to develop his analysis of radial groundwater flow. Ogata and Banks refer to Carslaw and Jaeger's work on heat diffusion to solve the longitudinal dispersion of solutes in flowing groundwater. The hydrogeologist is thus admirably equipped with the necessary conceptual models, mathematical tools and computer codes to become a practising thermogeologist.*

Introduction

It has pressed on my mind, that essential principles of Thermodynamics have been overlooked by … geologists
William Thomson, Lord Kelvin (1862)

The American psychologist Abraham Maslow (1908-1970) proposed a hierarchy of human needs (Maslow, 1943), often represented in the form of a pyramid. Maslow argued that the lowest levels of the pyramid (the fundamental human needs) must be satisfied before the individual can progress towards any form of self-actualisation, system of aesthetics and morals or (in short) happiness. Maslow includes amongst the most fundamental needs, in addition to food and oxygen, water and regulation of temperature. We, as geologists, are responsible for understanding and resourcing these basic needs from the subsurface environment - the hydrogeologist for groundwater and the thermogeologist for heat and "coolth" (Banks, 2012).

The term thermogeology is deliberately chosen (Banks, 2012) to provoke an awareness of the close analogy between the subsurface behaviour of water and of heat. Indeed, Banks (2012) has defined thermogeology as "the study of the occurrence, movement and exploitation of low enthalpy heat in the relatively shallow geosphere." We term a body of rock or sediment that possesses adequate permeability and storage properties to permit the economic abstraction of groundwater, an aquifer. Similarly, we can term a geological unit with a sufficient thermal conductivity and storage properties to permit the economic exploitation of subsurface heat, an aestifer (Latin aestus = heat, ferre = to carry or transmit). The word does, in fact, have an ancient pedigree: Virgil, Marcus Cicero and Lucretius all used to the term aestifer to describe, variously, the star Sirius and the constellation Cancer as the harbingers of summer heat, and the sun's rays as the bearers of heat (Possanza, 2001; Banks, 2009a).

The similarities between the subsurface behaviour of water and heat all have their origin in two physical principles:

1. Firstly, the ideal (non-turbulent) behaviour of groundwater and the conductive flow of heat both obey equations (Anderson, 2007) in which the flux is proportional to a potential gradient, and where the constant of proportionality is an intrinsic property of the matter (porous medium or thermal conductor). Idealised groundwater flow obeys the Darcy equation:

$$\vec{q} = -K\nabla h \quad (1)$$

or, in one dimension:

$$Q = -K_x A \frac{dh}{dx} \quad (2)$$

where: K is the hydraulic conductivity tensor (K_x = hydraulic conductivity in the x-direction); typically in m s^{-1}.
q is the groundwater flux (volume per time per unit cross-sectional area); typically in m s^{-1}.
h is groundwater head; typically in m.
A is cross sectional area; typically in m^2.
Q = groundwater flow rate; typically in m^3 s^{-1}.
Similarly, the conduction of heat obeys Fourier's Law:

$$\vec{q} = -\lambda\nabla\theta \quad (3)$$

or, in one dimension

$$Q = -\lambda_x A \frac{d\theta}{dx} \quad (4)$$

where λ is the thermal conductivity tensor (λ_x = thermal conductivity in the x-direction); typically in W m^{-1} K^{-1}
q is the heat flux per time per unit cross-sectional area; typically in J s^{-1} m^{-2} or W m^{-2}.
θ is temperature; typically in K.
A is cross sectional area; typically in m^2.
Q = heat flow rate; typically in J s^{-1} or W.

Thus, water flows down a potential gradient. Groundwater *head* (which combines pressure potential and gravitational potential, i.e. elevation) is the term we use to describe this potential. In the case of heat, *temperature* is the parameter that we use to describe the thermal potential. As regards the constant of proportionality, *hydraulic conductivity* in natural rocks and sediments spans maybe nine orders of magnitude; hence, we have *aquitards* and *aquifers*. *Thermal conductivity* has a very narrow range in rocks and sediments, typically between 1 and 5 W m^{-1} K^{-1}: thus most geological materials are potential *aestifers*.

Other Laws are also analogous to Darcy's and Fourier's Laws: namely, Fick's Law describing chemical diffusion of solutes along a concentration gradient, and Ohm's Law describing the flow of charge along an electrical potential gradient (Anderson 2007).

Jean-Baptiste Joseph Fourier's *On the Propagation of Heat in Solid Bodies* was delivered in around 1807; his *Théorie analytique de la chaleur* was published in 1822, some years after he had been appointed Professor at L'École Polytechnique in Paris (around 1795). It is interesting that Henry Darcy enrolled at L'École Polytechnique in 1821, at a time when Fourier's influence must still have been palpable. Darcy's own formula was described in his 1856 work *Les fontaines publiques de la ville de Dijon* (Figure 1).

2. The second physical principle is that both heat and water in the ground are essentially conservative. Thus, if we ignore the fact that small amounts of water may be lost by evaporation, or lost and gained during chemical reactions, we can write the three-dimensional groundwater flow equation (for an isotropic aquifer):

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{S_s}{K}\frac{\partial h}{\partial t} \quad (5)$$

where t is time (s) and S_s is specific storage (typically in m^{-1}).

Similarly, for heat conduction in the ground, and ignoring internal heat generated by radioactive decay or chemical reactions and any heat consumed or released by phase changes (and neglecting any convective transfer of heat, which will be considered in the section on Advection), we can write, for an isotropic aestifer:

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} = \frac{S_{VC}}{\lambda}\frac{\partial \theta}{\partial t} \quad (6)$$

We thus see that volumetric heat capacity, S_{VC} (typically in J m^{-3} K^{-1}) is the thermogeological analogue of groundwater storage coefficient. We can also understand that we should be able to use the same (or very similar) modelling approaches for both groundwater flow and subsurface heat flow.

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Fig. 1 - Heroes of thermogeology and hydrogeology: (a) Jean-Baptiste Joseph Fourier (1768-1830) and (b) Henry Philibert Gaspard Darcy (1803- 1858). Public domain images from Wikimedia Commons (http://commons.wikimedia.org).

Fig. 1 - Grandi della termogeologia e idrogeologia: (a) Jean-Baptiste Joseph Fourier (1768-1830) e (b) Henry Philibert Gaspard Darcy (1803- 1858). Immagini di pubblico dominio da Wikimedia Commons (http://commons.wikimedia.org).

Exploiting Heat and Groundwater using Pumps

It is common knowledge that water flows downhill, or from high pressure to low pressure (i.e. from areas of high head to low head). This principle is implicit in Darcy's Law (above). Groundwater naturally occurs at low head - several metres below ground level. Yet human beings require water at high head - such that it can be distributed from water towers or pressure tanks to household pipe systems. Thus, in order to exploit groundwater, we need two things:

1. An opening into the ground which allows the efficient transfer of groundwater from the aquifer to the opening. The opening may be vertical (a borehole or well) or horizontal (an infiltration gallery or qanat).
2. A *pump* which transfers water from a locus of low head (the aquifer) to one of high head (a water tower or pressure tank), from which it can be distributed (Figure 2).

The pump, of course, needs an input of energy to create a pressure differential to overcome the head difference. This energy input may be mechanical (a hand pump), hydraulic (a hydraulic ram) or electrical.

Likewise, it is common knowledge that "heat won't flow from a colder to a hotter". Your hot espresso macchiato never gets hotter, and your chilled vodka martini never gets any cooler, by leaving them out at ambient air temperature. Heat in the shallow subsurface typically occurs at low temperatures, approximately equal, or slightly greater than, average air temperature (Rambaut, 1900). Thus, ground temperatures of 10-12°C are common in the United Kingdom, with maybe 11-16°C being typical of Italy. Yet we like our buildings to be at a comfortable temperature of around 20°C. Thus, in order to exploit shallow subsurface heat, we need two things:

1. An opening into the ground which allows the efficient transfer of heat from the aestifer to the opening. The opening may be vertical (a heat exchanger in a borehole) or horizontal (a heat exchanger in a trench). See Geotrainet (2011a,b) and Banks (2012).
2. A *pump* which transfers heat from a locus of low temperature (the aestifer) to one of high temperature (a domestic space heating system), from which it can be distributed.

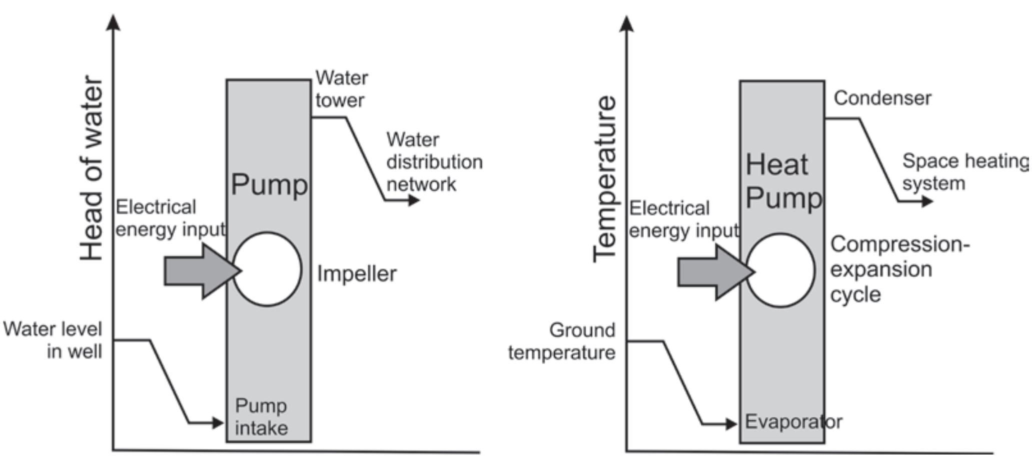

Fig. 2 - Conceptualisation of the analogies between an electrical centrifugal water pump and an electrical compression-expansion cycle heat pump.

Fig. 2 - Concettualizzazione delle analogie tra una pompa centrifuga elettrica per acqua e una pompa di calore elettrica a ciclo compressione-espansione.

The thermal pump is called, simply, a *heat pump.* It requires an input of energy to create a temperature differential between its cold thermal "inlet" (a heat exchanger called the evaporator) and its hot "outlet" (a heat exchanger called the condenser). The energy is used to power a refrigerant in a compression-expansion cycle: the input can be purely mechanical energy used to power a compressor (as in the first heat pumps constructed by Jacob Perkins in 1834) or electricity (as in most modern heat pumps). One can even find heat pumps that use high temperature heat to drive the refrigerant cycle (so-called absorption heat pumps). The first proposed use of the heat pump for space heating was by William Thomson, Lord Kelvin, Professor of the University of Glasgow, in 1852, in order to heat buildings at Queens College, Belfast (Thomson, 1852). The first patent on a ground-coupled heat pump was taken out by the Swiss Heinrich Zoelly in 1912 (Figure 3). The first ground-coupled heat pumps were probably not in use before the 1930s.

A hydrogeologist understands that the efficiency of a pump depends on the head difference between the input and the outlet. Depending on the type of pump, the greater the head difference up which the water must be pumped, the greater the power required or the smaller the pumped water quantity. Thus, the efficiency of pumping (water pumped divided by energy input) decreases as the pumping head increases. The relationship between these three quantities can usually be found in a set of *pump curves* provided by the manufacturer.

In a heat pump, the efficiency or *coefficient of performance* (COP, defined as the ratio between the heat output and the electrical energy input) depends on the temperature difference between the cool heat source and the warm heat sink. The greater the difference, the lower the COP. The relationships are again defined in a series of heat pump curves provided by the manufacturer. An efficient heat pump system will have a relatively high ground temperature (the heat source) and a low temperature space heating system (e.g. an underfloor waterborne heating system, maybe running at 35°C). In such cases, COP values of around 4 may be achievable. If the ground is cold and the space heating system very hot, the efficiency may drop to uneconomic values (COP of 3 or less).

Types of ground source heat pump system

There are two main methods (Figure 4) of coupling the evaporator (or condenser) of a heat pump to the ground so that heat can be extracted (for space heating) or rejected (for dehumidification or air conditioning).

The closed loop system

The *closed loop* system involves installing a heat exchanger in the ground, either vertically, within a borehole, or horizontally, within a backfilled trench. Often, the heat exchanger is simply a loop of high-density polyethene pipe, through which a *heat transfer fluid* (water, or a solution of anti-freeze in water) is circulated. The chilled heat transfer fluid leaves the evaporator (at temperatures of maybe as low as -3°C), it circulates down the borehole and heat flows by conduction from the warmer ground to the heat transfer fluid. The heat transfer fluid absorbs this heat and conveys it back to the surface. The heat transfer fluid, now at a temperature of maybe 0°C, enters the heat pump evaporator, the heat is extracted from the fluid and the re-chilled fluid returns down the heat exchange loop. In such a case, we would say that the average heat transfer flu-

Fig. 3 - Founding fathers of the heat pump: (a) Jacob Perkins (1766-1849), the first successful constructor of a heat pump, in an 1826 portrait by Thomas Edwards, appearing in the Boston Monthly Magazine; (b) William Thomson, Lord Kelvin (1824-1907), who first proposed space-heating with a heat pump; (c) Heinrich Zoelly (1862–1937), who patented the ground source heat pump. All images are believed to be public domain.

Fig. 3 - Padri fondatori delle pompe di calore: (a) Jacob Perkins (1766-1849), il primo costruttore di una pompa di calore, in un ritratto del 1826 di Thomas Edwards, pubblicato sul Boston Monthly Magazine; (b) William Thomson, Lord Kelvin (1824-1907), che per primo ha proposto il riscaldamento di ambienti tramite pompa di calore; (c) Zoelly Heinrich (1862-1937), che ha brevettato la pompa di calore geotermica. Tutte le immagini sono da ritenersi di dominio pubblico.

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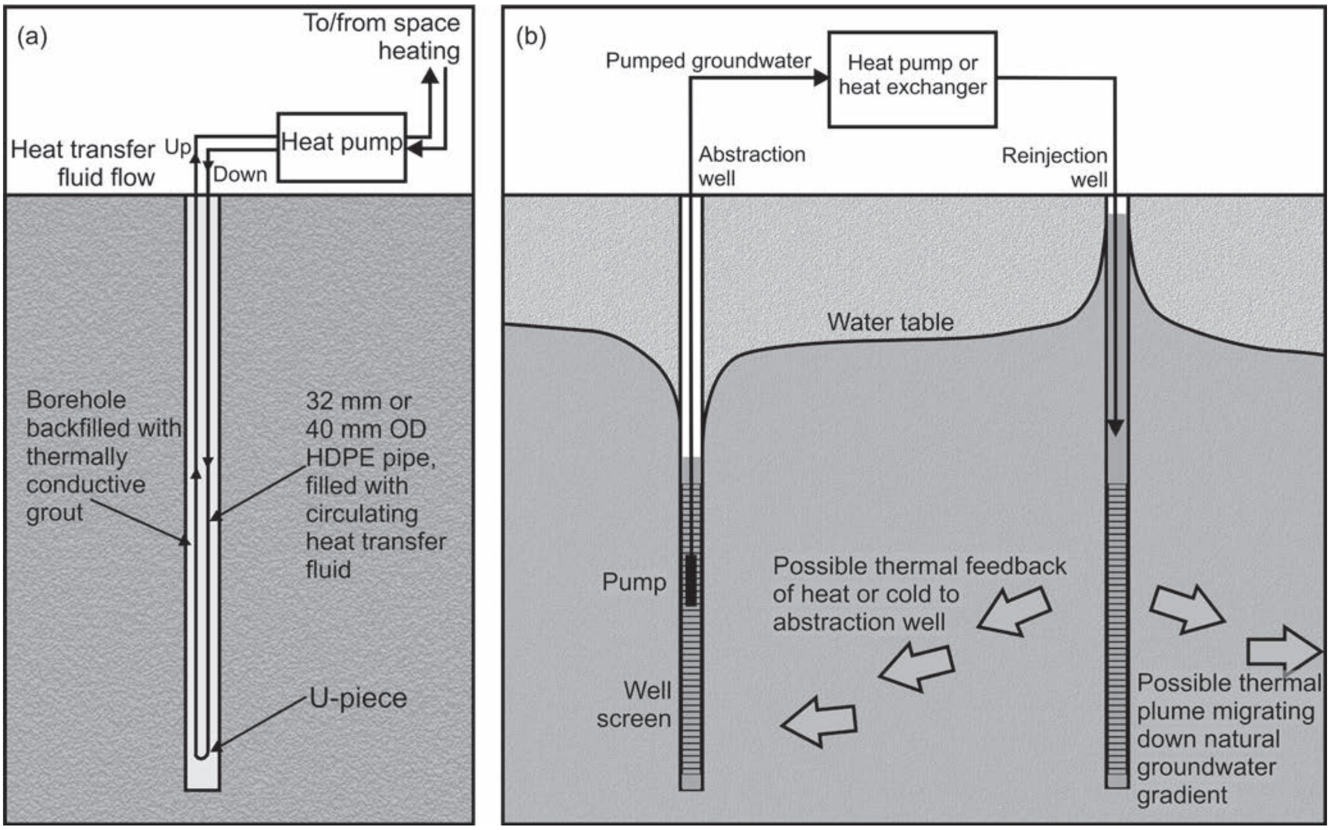

Fig. 4 - Schematic diagrams of (a) a closed-loop borehole heat exchanger ground source heat pump system and (b) a well-doublet open-loop ground source heating or cooling system. HDPE = high density polyethene, OD = outside diameter.

Fig. 4 - Diagrammi schematici di (a) un impianto a pompa di calore geotermica con scambiatore a circuito chiuso e (b) un impianto a due pozzi a circuito aperto, per riscaldamento o raffrescamento. HDPE = polietilene ad alta densità, OD = diametro esterno.

id temperature is (0°C - 3°C)/2 = -1.5°C. The colder the heat transfer fluid, the more heat is extracted from the ground, but the less efficiently the heat pump performs. (Similarly, in a water well, the greater the drawdown, the more water is extracted from the aquifer, but the more energy is expended in pumping the water to the surface). In a *borehole heat exchanger*, the *ground loop* is generally 32 mm or 40 mm OD HDPE pipe and a thermal contact with the borehole walls is usually achieved by sealing the borehole with a grout of high thermal conductivity and low hydraulic conductivity.

The open loop system

The *open loop* system involves drilling a normal water well and using a submersible pump to extract groundwater (which will be at the same temperature as the ground - maybe 12°C in the UK). This groundwater enters the evaporator of the heat pump (or, more usually, a heat exchanger thermally coupled to the evaporator) and heat is extracted directly from the groundwater flow. The “thermally spent” groundwater must now be disposed of - maybe to a sewer or drain, to the sea or to a river. It is common, however, for regulatory authorities to insist on the spent water being re-injected back into the aquifer via one or more injection wells, forming an abstraction-injection well doublet. The injected water is cold if the system is being used for space heating and is warm if it is being used for space cooling. Re-injection maintains groundwater resources and heads, but runs the risk of hydraulic and thermal ‘feedback’ to the abstraction well (Banks 2009b), or of a thermal plume of warm or cold groundwater migrating down the hydraulic gradient and impacting other users (Banks 2011).

Radial heat conduction to borehole heat exchangers

When a closed-loop borehole heat exchanger is turned on, the heat transfer fluid is chilled and forms a low-temperature line-sink. Radial conduction of heat is induced from the surrounding rocks and a zone of depressed temperature forms around the borehole, which expands with time. Horatio Scott Carslaw was the first person to find a solution to the time-dependent radial flow of heat from a line source (or to a line sink), by envisaging an infinite thermally conductive plate sandwiched between two insulators (Figures 5, 6). Carslaw was not a geologist, but the geologist will immediately recognise this as the geometry of a vertical borehole heat exchanger in geological sequence. Carslaw’s mathematical solution for a line sink was not simple, but it was rigorous:

$$\theta_0 - \theta = \frac{Q}{4\pi\lambda D} E(u) = \frac{q'}{4\pi\lambda}\left[-\gamma - \ln(u) - \sum_{n=1}^{\infty}(-1)^n \frac{u^n}{n.n!}\right] \quad (7)$$

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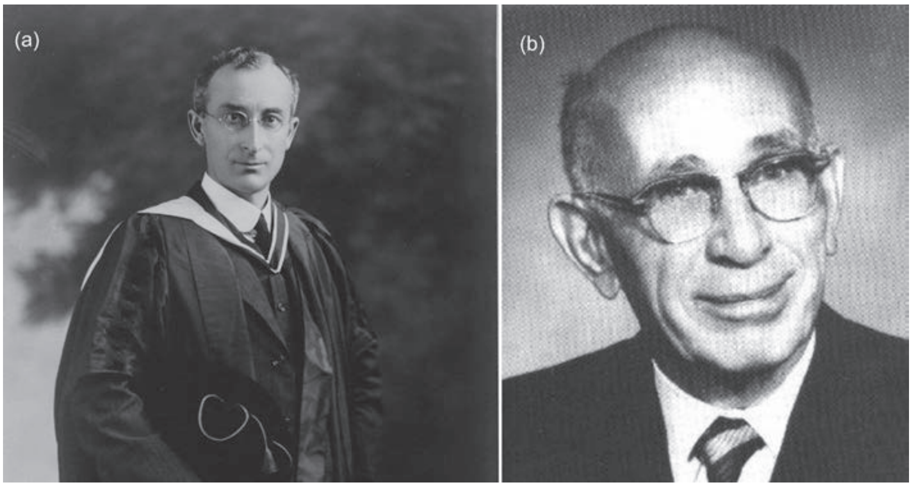

Fig. 5 - Time-dependent radial flow theorists: (a) Horatio Scott Carslaw (1870-1954), a lecturer at the University of Glasgow and subsequently Professor at the University of Sydney (reproduced by kind permission of Prof. Robert Frank Carslaw Walters of the University of Insubria, Como, Italy), and (b) Charles V. Theis (1900-1987), from public domain USGS document, edited by Clebsch (1994).

Fig. 5 - Teorici del flusso radiale dipendente dal tempo: (a) Horatio Scott Carslaw (1870-1954), docente presso l'Università di Glasgow e successivamente Professore presso l'Università di Sydney (riprodotta per gentile concessione del Prof. Robert Frank Carslaw Walters dell'Università degli Studi dell'Insubria, Como, Italia), e (b) Carlo V. Theis (1900-1987), da documento di pubblico dominio USGS, a cura di Clebsch (1994).

where:

$$u = r^2 S_{VC} / (4\lambda t) \tag{8}$$

θ_0 and θ are the temperatures (K) at any radial distance r (m) from the line sink at time t = 0 and time t (both in s), respectively.
Q is the heat extraction rate from the line sink (W).
q' is the heat extraction rate per unit depth of the line sink (W m^{-1})
D is the thickness of the conductive slab (depth of the line sink) (m).
γ is Euler's constant

For small values of u, this equation can be simplified to a logarithmic form

$$\theta_0 - \theta = \frac{q'}{4\pi\lambda}\left[\ln\left(\frac{4\lambda t}{r^2 S_{VC}}\right) - 0.5772\right] \tag{9}$$

These equations should look very familiar to the hydrogeologist. Charles V. Theis, when faced with the problem of deducing the time-dependent radial flow of groundwater to a well, was poor enough at mathematics and humble enough to ask advice from his friend, Clarence Lubin of the University of Cincinnati. Theis had a clear conceptualisation of the problem, however. He wrote to Lubin in 1934:

"We have exact analogies in ground water theory for thermal gradient, thermal conductivity, and specific heat. I think a close approach to the solution of some of our problems is probably already worked out in the theory of heat conduction. Is this problem in radial flow worked out?" (Freeze, 1985).

Lubin in turn referred to Carslaw's (1921) work and the Carslaw equation formed the basis for Theis's equation for calculating drawdown:

$$s = h_0 - h = \frac{Q}{4\pi KD} E(u) \tag{10}$$

where:

$$u = r^2 S / (4Tt) \tag{11}$$

which was simplified, when u is small, by Cooper and Jacob (1946) to

$$s = \frac{Q}{4\pi T}\left[\ln\left(\frac{4Tt}{r^2 S}\right) - 0.5772\right] \tag{12}$$

where:
D = aquifer thickness (m).
T = transmissivity (m^2 s^{-1})
Q = water discharge rate (m^3 s^{-1})
s = drawdown (m)
h_0 and h are the groundwater heads (m) at any radial distance r (m) from the line sink at time t = 0 and time t (both in s), respectively

Theis (1935) explicitly acknowledged the debt that his equation owed to analogies with heat conduction theory in general and Carslaw's line source equation in particular.

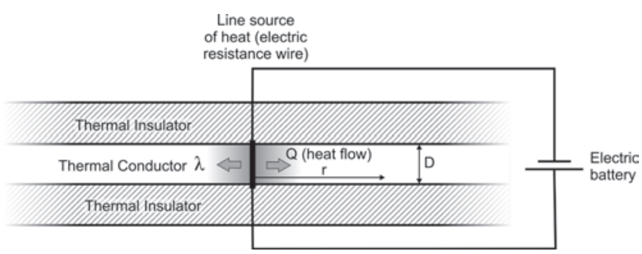

Fig. 6 - A conceptualisation of Horatio Scott Carslaw's line heat source in an infinite conducting plate. Slightly modified after Banks (2012) (© John Wiley & Sons, Chichester 2012).

Fig. 6 - Una concettualizzazione della fonte di calore lineare in una piastra conduttiva infinita di Horatio Scott Carslaw. Leggermente modificato da Banks (2012) (© John Wiley & Sons, Chichester 2012).

Well and borehole efficiency

In hydrogeology, it is of interest to be able to predict the drawdown in a pumping well. In an ideal well, this can be done by setting the radius r to r_b, the borehole or well radius, in equations (10)-(12). In non-ideal wells, there will usually be an additional component of drawdown ascribable to the hydraulic resistance of the well structure itself and to factors such as turbulent (non-Darcian) flow. This additional hydraulic resistance is typically regarded as having linear and non-linear (with respect to discharge rate, Q, typically in $m^3\ s^{-1}$) components (see also Forchheimer, 1914). Thus, we can write (Bierschenk, 1963; Hantush, 1964):

$$s_b = BQ + CQ^2 \qquad (13)$$

where
s_b = drawdown in the pumping well or borehole; typically in m.
C = a non-linear well-loss constant; typically in $s^2\ m^{-5}$.

$$B = \frac{E(u)}{4\pi KD} + B' \qquad (14)$$

B' = any linear well-loss constant; typically in $s\ m^{-2}$.

With a closed loop borehole heat exchanger, the situation is similar. We can obtain the thermal drawdown (θ_0-θ_b) in the heat transfer fluid by setting r to r_b, the radius of the heat transfer borehole, and adding a term for borehole thermal resistance. Fortunately, this can be considered approximately linear with respect to heat extraction rate and encompasses many different components of thermal resistance (conductivity of grout and plastic pipe walls, thermal short-circuiting between upflow and downflow pipe shanks) in a single term. Thus, equations (7)-(9) become:

$$\theta_0 - \theta_b = \frac{Q}{4\pi\lambda D}E(u) + \frac{QR_b}{D} \approx \frac{q'}{4\pi\lambda}\left[\ln\left(\frac{4\lambda t}{r^2 S_{VC}}\right) - 0.5772\right] + q' R_b \qquad (15)$$

where:
θ_b is the average heat transfer fluid temperature (K) in the closed loop borehole (average of upflow and downflow temperatures)
R_b = borehole thermal resistance constant (usually in $K\ m\ W^{-1}$).

This equation allows us to predict the fluid temperatures in the borehole heat exchanger at any time t, provided we have some idea of the values of θ_0, S_{VC}, R_b and λ. In designing a borehole heat exchanger, it is common to increase the value of D (the length of borehole heat exchanger) until a reasonable value of θ_b, which is not too low for efficient heat pump performance, is achieved.

Hydraulic and thermal pumping tests

In hydrogeology, the values of transmissivity (and, sometimes, storage) can be obtained by an inverse solution of equations (10) to (12). The borehole is pumped at a constant rate Q and the aquifer response is measured in terms of drawdown with respect to time. In Cooper and Jacob's (1946) analysis method, drawdown is plotted against the logarithm of time. Equation (12) predicts that this should be a straight line, from whose gradient a value of transmissivity can be derived.

In a so-called thermal response tests (TRT - Gehlin, 2002, Banks, 2012), heat is extracted from (or more commonly, injected to) a closed loop borehole heat exchanger at a constant rate. The flow rate and upflow and downflow temperatures of the heat transfer fluid are measured (the difference between these temperatures, together with the flow rate give the applied heat power, while the average of the temperatures is defined as θ_b). After several hours (c. 10 hours, in many cases), the logarithmic approximation of equation (15) becomes valid. The thermal displacement (θ_b - θ_0) is plotted against the logarithm of time. From the gradient of the response, the thermal conductivity of the ground can be deduced and, provided the volumetric heat capacity of the ground, S_{VC}, is approximately known, the borehole thermal resistance, R_b, can be derived from the y-intercept.

Boundary conditions

So far, it appears that the analogy between Darcian groundwater flow and subsurface heat conduction is almost perfect. Are there any discrepancies at all? There are at least two minor differences in our conceptions of groundwater systems and subsurface heat systems.

1. We normally conceptualise an aquifer unit as having a constant groundwater head h with depth, corresponding with the elevation of the piezometric surface or the water table. We know, however, that the temperature of the ground varies with depth. Under natural, steady-state conditions in stable continental areas, it typically increases by 1 to 3 K per 100 m depth. However, as we are normally concerned, in thermogeology, with simulating the temperatures of the heat transfer fluid in a real borehole, it turns out that, for practical purposes (Eskilson, 1987), we can consider θ_0 or θ_b as simply being the average temperature along the entire length of the borehole. In fact, in hydrogeology, groundwater head usually *does* vary with depth in an aquifer (although we do not always consider this as much as we should). What we measure in a well is some kind of average head along the length of the well - so we are making the same simplification as in thermogeology.
2. In confined aquifers, the aquifer is normally conceptualised as having a no-flow boundary at its base and a no-flow boundary at its upper surface. In an unconfined aquifer, the upper boundary is normally simulated as having a constant recharge rate (or, at least, a pre-determined time-dependent recharge). These are all forms of Type 2 (Neumann) boundary conditions (Figure 7). A typical aestifer has a fixed heat flux (the geothermal heat flux of several tens of $mW\ m^{-2}$) entering from the

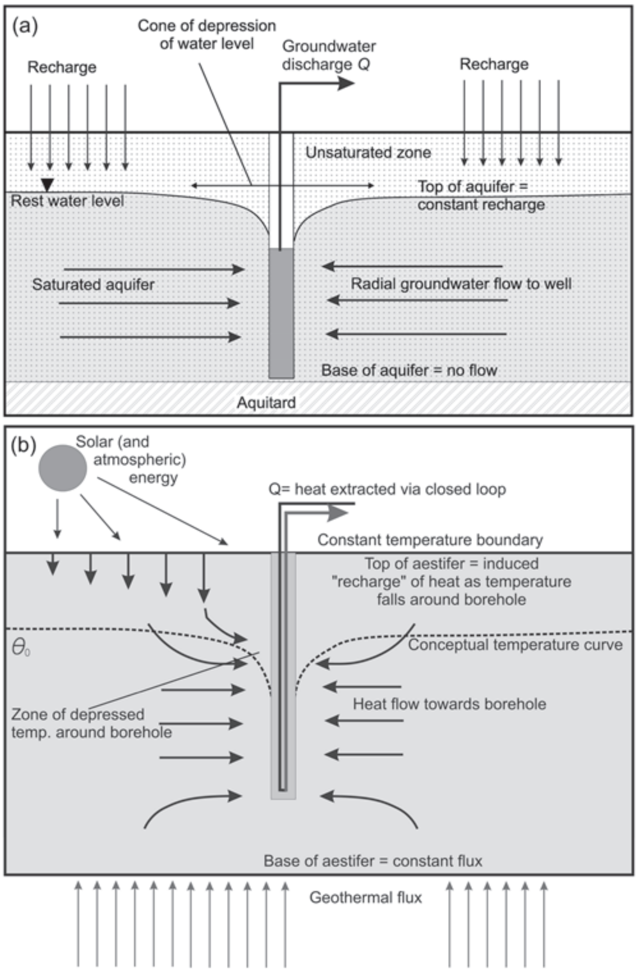

Fig. 7 - Comparison of conceptual models, with upper and lower boundary conditions, for (a) a well in an unconfined aquifer and (b) a closed loop borehole heat exchanger in a typical aestifer. θ_0 is the undisturbed "rest" temperature in the aestifer. Slightly modified after Banks (2012) (© John Wiley & Sons, Chichester 2012).

Fig. 7 - Confronto di modelli concettuali, con condizioni al contorno superiori ed inferiori, per (a) un pozzo in un acquifero non confinato e (b) uno scambiatore di calore a circuito chiuso in una tipica roccia conduttiva. θ_0 è la temperatura indisturbata "a riposo" nella roccia conduttiva. Leggermente modificato da Banks (2012) (© John Wiley & Sons, Chichester 2012).

base (a Type 2 Neumann boundary condition). Its upper surface is more complex, however. The aestifer can lose heat to the atmosphere if it is warmer than the soil and air. At can also induce downward heat flow from the atmosphere/soil if it becomes colder. In most models, the upper boundary of the aestifer is thus simulated as a Type 1 Dirichlet boundary: i.e. a constant temperature boundary, corresponding to the annual average soil or air temperature over the long term.

When a well starts pumping from a Neumann-bounded aquifer, the cone of depression expands indefinitely, until at some point it intercepts a boundary of head-dependent (Type 1) character and is stabilised by, e.g., induced flow via the bed of a stream (see Theis, 1940 and Bredehoeft et al., 1982). When a borehole heat exchanger starts operating in an aestifer, the zone of depressed temperature does not expand indefinitely; at some point, the chilled aestifer will begin to induce a flow of heat from the soil and atmosphere, which will tend to stabilise the downward trend of temperature, ultimately approaching some kind of steady state. It may well take several decades for this stabilisation to become apparent, however, although Eskilson (1987) provides mathematical formulae for calculating the steady state condition.

In fact, the exact nature of heat transfer across the upper boundary of the aestifer is highly complex, involving absorption of solar and atmospheric radiation, back radiation, infiltration of heat with rainfall, evapotranspiration and conduction. Some of these processes are temperature-dependent, others are less so. The real boundary condition may not therefore be a perfect Dirichlet boundary - a mixed, or Cauchy-type boundary condition may be more appropriate - and further research on these heat transfer processes would be beneficial.

Advection of heat with groundwater flow

Many aestifers are also aquifers. Heat flow may, in these cases, be transported by advection with groundwater flow, as well as by pure conduction. This will normally be to the benefit of an operational borehole heat exchanger, as the groundwater will advect 'new' heat into the chilled zone around the borehole, tending to lead to a stabilisation of temperatures.

If we wish to simulate the advection of heat with groundwater flow into or away from the region of a closed loop borehole, or if we want to simulate the thermal feedback within an open loop well doublet or the movement of a thermal 'plume' of hot or cold water down-gradient from an injection well (Figure 4), we will typically wish to employ a 3-dimensional, coupled heat/flow, finite element code, such as FeFLOW (Trefry and Muffels, 2007) or similar. Two-dimensional approximations can be made, however (Banks, 2009b, 2011) and the hydrogeologist should be aware that the advection of heat with groundwater flow is directly analogous to the advection of a sorbed solute with groundwater flow.

The transport velocity of a solute is retarded, relative to groundwater flow, by a retardation factor R_f (>1), which is determined by the degree to which the solute is adsorbed to the surfaces of the aquifer matrix (i.e. the distribution coefficient of the solute between the absorbed form and the groundwater). The apparent average transport velocity of the sorbed contaminant (v_c) is given by

$$v_c = \frac{v}{R_f} = \frac{v_D}{R_f n_e} = -\frac{K}{R_f n_e}\frac{dh}{dx} \qquad (16)$$

where v is the average groundwater flow velocity; typically in m s^{-1} and v_D is the Darcy flux, typically in m s^{-1} (groundwater flow per unit cross-sectional area) = $v n_e$, where n_e is the effective porosity of the aquifer (dimensionless).

The full equation for the advective dispersion of a sorbed solute is

$$R_f \frac{\partial C}{\partial t} = \nabla(D\nabla C) - \nabla(vC) = \nabla(D\nabla C) - \nabla(\frac{v_D}{n_e} C) \quad (17)$$

where C is the concentration of the solute, and D is a dispersion coefficient (taking into account both molecular diffusion and hydrodynamic dispersion).

De Marsily (1986) notes that heat is retarded relative to groundwater flow, due to the fact that heat diffuses from the water into the solid matrix of the aquifer. If we assume that this diffusion is very fast and that the aquifer matrix almost instantaneously equilibrates with the groundwater temperature, De Marsily cites the equation for the advective transport of heat as

$$\frac{S_{VCaq}}{S_{VCwat}} \frac{\partial \theta}{\partial t} = \nabla(\frac{\lambda *}{S_{VCwat}} \nabla \theta) - \nabla(v_D C) \quad (18)$$

where S_{VCaq} and S_{VCwat} are the volumetric heat capacities of the bulk saturated aquifer material and water respectively, λ* is a modified thermal conductivity (or thermal dispersion factor) taking into account both thermal diffusion and hydrodynamic dispersion (see also the Chapter on "Transport of heat" in Parkhurst and Appelo 1999).

By analogy between the two equations, we can see that the thermal retardation relative to the Darcy velocity (v_D) is given by:

$$\frac{v_D}{v_{th}} = \frac{S_{VCaq}}{S_{VCwat}} \quad (19)$$

and the thermal retardation factor (R_{th}) relative to the average linear groundwater flow velocity (v) is given by:

$$R_{th} = \frac{v}{v_{th}} = \frac{S_{VCaq}}{n_e S_{VCwat}} \quad (20)$$

where v_{th} is the apparent average transport velocity of a heat front.

For a typical sandy aquifer, with n_e = 0.15, and S_{VCaq} =2.3 MJ m^{-3} K^{-1}, the retardation factor would be 3.65, given that S_{VCwat} = 4.2 MJ m^{-3} K^{-1}. The dispersion equation can be solved, for heat, in the same manner as for a solute, using numerical or analytical solutions, such as the 1-dimensional longitudinal case of Ogata and Banks (1961). It should be noted that the Ogata-Banks paper also draws on Carslaw's work (Carslaw and Jaeger, 1947) to find a conductive heat flow analogy to the groundwater case. It should be noted, however, that realistic simulations of heat transport in aquifers require 3-dimensional simulations: Banks (2011) notes that upward conduction of heat to the surface from an aquifer, and downward conduction of heat through an underlying aquitard, can be extremely effective mechanisms for attenuating an advected thermal plume.

Conclusion

The analogy between the behaviour of groundwater in an aquifer and the conductive behaviour of heat in an aestifer is almost perfect. The development of groundwater flow theory has consistently drawn upon thermal analogues to find mathematical solutions. The hydrogeologist is thus admirably equipped with the necessary conceptual models, mathematical tools and computer codes to become a practising thermogeologist. Many thermogeological problems can be conceptualised and understood by envisaging the hydrogeological analogue - or vice versa. For example, a hydrogeologist who has worked on artificial recharge projects will immediately understand the concept of underground thermal energy storage (UTES). In the former, surplus winter surface water may be treated and injected to an aquifer, where it can be stored until a time of water scarcity (summer, or a drought year), when it can be re-abstracted. Similarly, heat pumps can be used to "suck" surplus heat out of buildings in summer (thus providing space-cooling or air-conditioning) and to discharge it into the ground. A portion of this heat will be retained in the ground, resulting in elevated ground temperatures. In the winter, this heat can be re-abstracted for use in space heating. This practice can result in highly efficient system energy efficiencies.

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